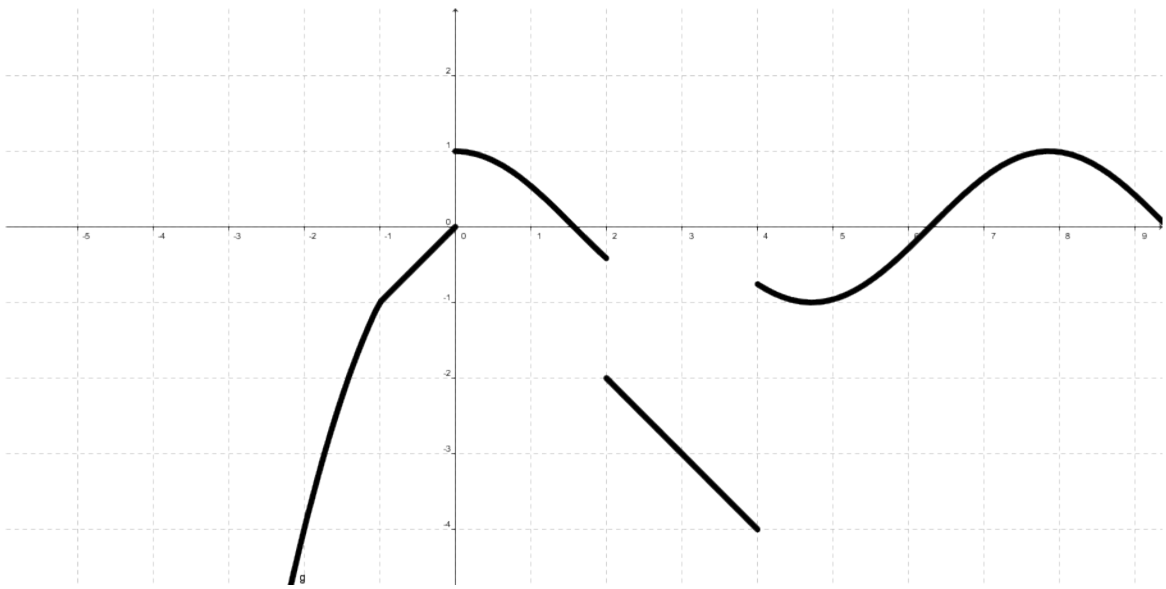
1 Limits

Highlights: Secant Line Approximation (light emphasis), Right and Left Hand Limits, Limits of Infinity, Computing Limits, Squeeze Theorem (light emphasis), Continuity, Piecewise Functions, L'Hopital's Rule (from later chapters, but involves limits)

1. Use the graph of $g(x)$ below to find the following limits. If the limit does not exist, explain why it does not exist.



$\lim_{x\to -1} g(x)$ $\quad$ $\lim_{x\to 0^-} g(x)$ $\quad$ $\lim_{x\to -\infty} g(x)$

$\lim_{x\to 0^+} g(x)$ $\quad$ $\lim_{x\to 2} g(x)$ $\quad$ $\lim_{x\to +\infty} g(x)$

2. Sketch a function $f(x)$ based on the following conditions. State where $f(x)$ is continuous, continuous from the right, and continuous from the left.

$\lim_{x\to -2^-} f(x) = 4$ $\quad$ $f(-2) = 4$ $\quad$ $\lim_{x\to 1^+} f(x) = -\infty$

$f(0) = 0$ $\quad$ $\lim_{x\to -\infty} f(x) = 2$

$\lim_{x\to -2^+} f(x) = -3$ $\quad$ $\lim_{x\to 1^-} f(x) = +\infty$ $\quad$ $\lim_{x\to +\infty} f(x) = 0$

3. Compute the following limits. Utilize *any and all techniques* learned this semester. If a limit does not exist, explain why.

$$\lim_{x \to 2} \frac{x^2+5}{2x+6}$$

$$\lim_{x \to -3} \frac{x}{x^2-9}$$

$$\lim_{x \to 5^+} \frac{x-4}{x^2-7x+10}$$

$$\lim_{x \to \pi/3} \frac{\tan^2 x \cot x + 1}{\sec x}$$

$$\lim_{x \to \infty} \frac{\sqrt{x-101}}{x+1}$$

$$\lim_{x \to -\infty} \frac{3x^5-5x^2+7}{20x^2+x-9}$$

$$\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$$

$$\lim_{x \to 0} (\csc x - \cot x)$$

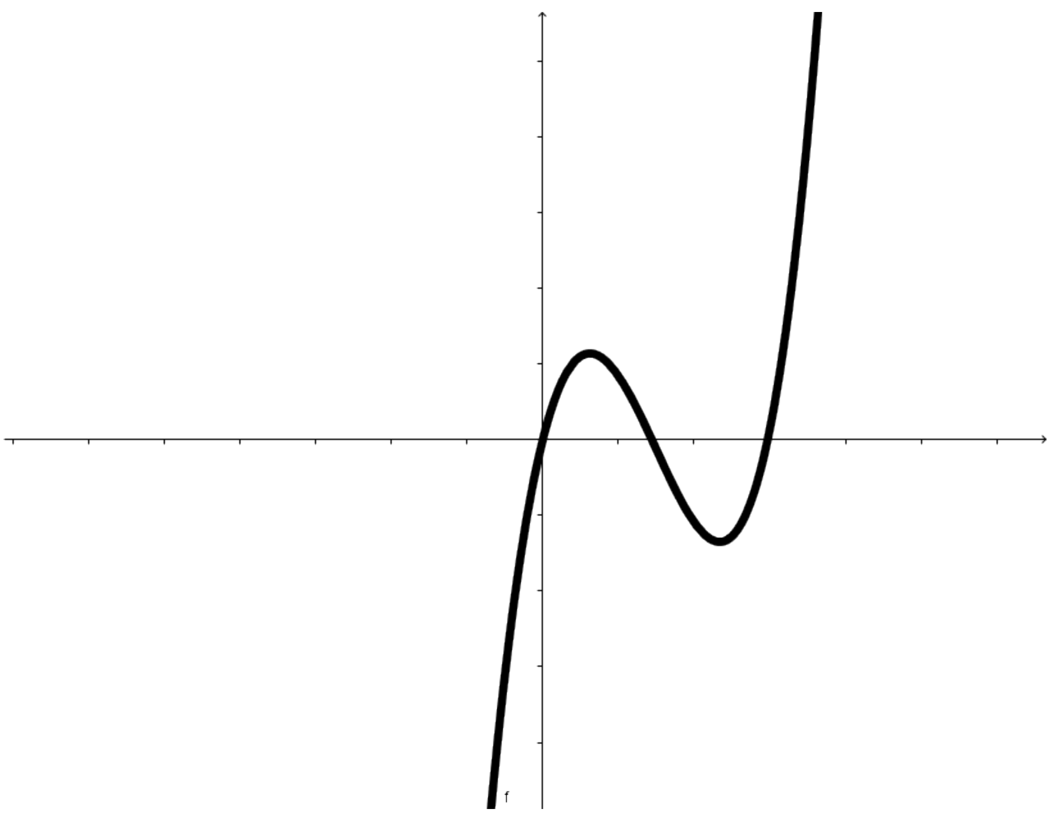
$$\lim_{x \to 0} \frac{\cos(mx) - \cos(nx)}{x^2}$$

2 Differentiation

Highlights: Definition of a Derivative, Sketching the Derivative of a function, Sum/Difference Rule, Power Rule, Product Rule, Quotient Rule, Chain Rule, Implicit Differentiation, Trigonometric Differentiation, Logarithmic and Exponential Differentiation

4. Find the derivative of $f(x) = x + \sqrt{x}$ by *using the definition of a derivative*. Check your answer by other methods.

5. Sketch the derivative of the function below.



6. Differentiate the following functions, find y', or dy/dx. Utilize *any and all techniques* learned in the course.

$$y = x^4 + 2x^3 + 3x^{3/2} + \frac{5}{x} + \ln(2x)$$

$g(t) = 3t^{5/3}\ln(t^2) + \sin t \cos t$

$3xy = -y^2\tan(x) + \dfrac{e^{xy}}{1-2x}$

$y = \dfrac{(\sqrt[3]{x+2})e^{x^4}(1-x^2)}{x-1}$

7. Find the tangent line to the curve $y = x^2\sqrt{x}$ at the point $(4, 32)$.

2.1 Applications of Differentiation

Important Problem Types: Filling Tanks, Marginal Cost, Exponential Growth/Decay, Half-Life, Related Rates, Optimization, Differentials, Linearization, Newton's Method (light emphasis)

8. A man starts walking north at 4 ft/s from a point P. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft east of P. At what rate are the people moving apart 15 minutes after the woman starts walking?

9. A drinking glass is in the shape of a inverted *frustrum* of a cone – a cone with its tip cut off. The drinking glass is being filled with water at a rate of 3 cm^3/s. If the top of the glass has a radius that is 1/3 the height of the glass and the bottom of the glass has a radius of 2 cm, how fast is the height of the water rising when the height of the water is 8 cm? The volume of a frustrum is $V = \frac{\pi h}{3}(R^2 + Rr + r^2)$ where r is the smaller radius and R is the larger radius.

10. Alex wants to build a rectangular addition for storage onto his house and wants to minimize the cost of the materials. After doing some research, he finds that the cost per square foot for each wall will be 20 dollars, the floor will be 25 dollars, and the ceiling will be 30 dollars. Alex wants the addition to be able to hold 1000ft^3 of material and the width must be twice the length. What dimensions minimize the cost?

11. Find Δy and dy for the given value of x and $dx = \Delta x$ of the function

$$y = 5x^4 - 3x^3 - 3x^2 - 4x, \ x = 1, \ dx = 0.25$$

3 Curve Sketching

Highlights: Critical Values/Absolute/Local Extrema, Intervals of Increase/Decrease, Points of Inflection, Intervals of Concavity, Curve Sketching, Light emphasis on Rolle's Theorem and the Mean Value Theorem

Use the following steps to obtain the graph of each function below:

1. Determine the Domain
2. Locate ALL intercepts
3. Determine any symmetry
4. Find any Asymptotes
5. Find the intervals of increase/decrease
6. Determine Local Max/Min Values
7. Find Points of Inflection and Intervals of Concavity
8. Sketch the curve

12. $y = \ln(x^2 - 3x + 2)$

13. $y = (x^2 - 3)e^{-x}$

4 Antidifferentiation/Integration

Highlights: Riemann sums using left, right, and middle endpoints, Indefinite Integration, Definite Integration, U-Substitution, Areas between curves (x and y areas), Volumes of revolution (x and y axes), Volumes of revolution by Cylindrical Shells

14. Approximate the area under the following curve on $[-4,4]$ using 4 approximating rectangles and (a) Left Endpoints (b) Right Endpoints (c) Midpoints

$$f(x) = x^2 + 4x + 6$$

15. Use integration to compute the exact area under the curve in Exercise 14.

16. Compute each integral using *any and all* techniques from the course.

$$\int_0^6 \left(x^2 + 5x - 2 - x^{3/2}\right) dx$$

$$\int \frac{5 - 2t^4 + te^t}{t}\, dt$$

$$\int_0^{\pi/4} \left(3\sec^2\theta - \sin\theta + \frac{\cot^2(\theta)\sin^2(\theta)}{1-\sin^2\theta}\right) d\theta$$

$$\int \frac{x+1}{\sqrt{x-1}}\,dx$$

$$\int (\sqrt{x})\sin(1+x^{3/2})\,dx$$

17. Let $f(x) = \int_0^x \frac{x^3+5x^2-9x-45}{3+t}\,dt$**. Determine the values of** c **such that the conclusion of the Mean Value Theorem holds on the interval** $[0,2]$**.**

18. Find the area between the curves. Hint: one point of intersection is $x = 2$**; find the other intersection point.**

$$y = x^3 - 2x^2 + 3,\ y = x^2 - 1$$

19. Find the volume generated by revolving the area enclosed by $y^2 = x$ **and** $x = 2y$ **(a) about the** y**-axis (b) about the** x**-axis (c) about the line** $x = -2$ **(d) about the line** $y = -1$**.**

20. A cylindrical tank that is 12 feet high and has a radius of 4 feet that is standing on end (circle side against the ground) has some water in it. An engineer decides to empty the tank of the water and pump the water to a large secondary holding tank that is 4 feet above the top of the cylindrical tank. If the water is being pumped at a rate of $6\ ft^3/min$, how much work will it take to pump all of the water out of th tank if 8 feet of water is still in the tank? (Assume that gravity is acting on the water and the air in the cylindrical tank is at the top of the tank.)

21. Determine the values c such that $f_{ave} = f(c)$ on $[-4,3]$ for $f(x) = x^2 - 3x + 1$